Operators and Matrices
Volume 14, Number 2 (2020), 295–304
doi:10.7153/oam-2020-14-23

SEMIGROUP GENERATIONS OF UNBOUNDED BLOCK OPERATOR MATRICES BASED ON THE SPACE DECOMPOSITION

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Abstract. This paper deals with the problem for unbounded block operator matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

with natural domain to generate C_0 semigroups, based on the space decomposition. By describing the spectral inclusion relations between the numerical range of M and its inner entries, using the quadratic complements of M, some necessary and sufficient conditions for M to generate C_0 semigroups are given.

Mathematics subject classification (2010): 47D03, 47B99, 37K99.
Keywords and phrases: Unbounded block operator matrix, generator, C_0 semigroup, space decomposition.

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Paper OaM-14-23