

## ON THE SPECTRUM OF THE SYLVESTER–ROSENBLUM OPERATOR ACTING ON TRIANGULAR ALGEBRAS

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*Abstract.* Let  $\mathcal{A}$  and  $\mathcal{B}$  be algebras and  $\mathcal{M}$  be an  $\mathcal{A}$ - $\mathcal{B}$ -bimodule. For  $A \in \mathcal{A}$ ,  $B \in \mathcal{B}$ , we define the Sylvester-Rosenblum operator  $\tau_{A,B} : \mathcal{M} \rightarrow \mathcal{M}$  via  $\tau_{A,B}(M) = AM + MB$  for all  $M \in \mathcal{M}$ . We investigate the spectrum of  $\tau_{A,B}$  in three settings, namely: (a) when  $\mathcal{A} = \mathcal{B} = \mathcal{T}_n(\mathbb{F})$ , the set of upper-triangular matrices over an algebraically closed field  $\mathbb{F}$  and  $\mathcal{M} \subseteq \mathbb{M}_n(\mathbb{F})$ ; (b) when  $\mathcal{A} = \mathcal{B} = \mathcal{M}$  is a unital triangular Banach algebra; and (c), when  $\mathcal{M} = \mathcal{T}(\mathcal{N})$  is the nest algebra associated to a nest  $\mathcal{N}$  on a complex, separable Hilbert space and  $\mathcal{A} = \mathcal{B} = \mathcal{CI} + \mathcal{K}(\mathcal{N})$  consists of the unitization of the algebra of compact operators in  $\mathcal{T}(\mathcal{N})$ .

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