INEQUALITIES FOR WEIGHTED GEOMETRIC MEAN IN HERMITIAN UNITAL BANACH ∗-ALGEBRAS VIA A RESULT OF CARTWRIGHT AND FIELD

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Abstract. Consider the quadratic weighted geometric mean

\[ x \boxtimes_{\nu} y := \left| \left| y x^{-1} \right| x \right|^2 \]

for invertible elements \( x, y \) in a Hermitian unital Banach ∗-algebra and real number \( \nu \). In this paper, by utilizing a result of Cartwright and Field, we obtain various upper and lower bounds for the positive difference

\[ (1 - \nu) |x|^2 + \nu |y|^2 - x \boxtimes_{\nu} y, \]

where \( \nu \in [0,1] \), under various assumptions for the elements involved. Applications for the classical weighted geometric mean

\[ a^{\nu} b := a^{1/2} \left( a^{-1/2} b a^{-1/2} \right)^{\nu} a^{1/2} \]

of positive elements \( a, b \) that satisfy the condition \( 0 < ka \leq b \leq Ka \) for certain numbers \( 0 < k < K \), are also given.


Keywords and phrases: Weighted geometric mean, weighted harmonic mean, Young’s inequality, operator modulus, arithmetic mean-geometric mean inequality, Hermitian unital Banach ∗-algebra.

REFERENCES


