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EIGENVALUES OF DISCRETE STURM–LIOUVILLE PROBLEMS WITH SIGN–CHANGING WEIGHT AND COUPLED BOUNDARY CONDITIONS

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Abstract. In this paper, we study the eigenvalues of discrete Sturm-Liouville problems with sign-changing weight and coupled boundary conditions. The exact number (including multiplicity) of the real eigenvalues is obtained. The number of positive eigenvalues is equal to the number of positive elements in the weight function, and the number of negative eigenvalues is equal to the number of negative elements in the weight function. Meanwhile, the interlacing properties of these eigenvalues are also obtained as the parameter varies. These results extend the relevant existing results of discrete left-definite and right-definite Sturm-Liouville problems with coupled boundary conditions.

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