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ON POWER DRAZIN NORMAL AND DRAZIN QUASI–NORMAL HILBERT SPACE OPERATORS

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Abstract. A Drazin invertible Hilbert space operator $T \in B(\mathscr{H})$, with Drazin inverse T_d, is (n,m)-power D-normal, $T \in [(n,m)DN]$, if $[T_d^n, T^{*m}] = T_d^n T^{*m} - T^{*m} T_d^n = 0$; T is (n,m)-power D-quasinormal, $T \in [(n,m)DQN]$, if $[T_d^n, T^{*m}T] = 0$. Operators $T \in [(n,m)DN]$ have a representation $T = T_1 \oplus T_0$, where T_1 is similar to an invertible normal operator and T_0 is nilpotent. Using this representation, we have a keener look at the structure of $[(n,m)DN]$ and $[(n,m)DQN]$ operators. It is seen that $T \in [(n,m)DN]$ if and only if $T \in [(n,m)DQN]$, and if $[T,X] = 0$ for some operators $X \in B(\mathscr{H})$ and $T \in [(1,1)DN]$, then $[T_d^*, X] = 0$. Given simply polar operators $S,T \in [(1,1)DN]$ and an operator $A = \begin{pmatrix} T & C \\ 0 & S \end{pmatrix} \in B(\mathscr{H} \oplus \mathscr{H})$, $A \in [(1,1)DN]$ if and only if C has a representation $C = 0 \oplus C_{22}$.

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