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SCHRÖDINGER OPERATORS WITH POTENTIAL WAVEGUIDES ON PERIODIC GRAPHS

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Abstract. We consider discrete Schrödinger operators with periodic potentials on periodic graphs perturbed by guided positive potentials, which are periodic in some directions and finitely supported in other ones. The spectrum of the unperturbed operator is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. It is known that the spectrum of the perturbed operator consists of the spectrum of the unperturbed one and the additional guided spectrum, which is also a union of a finite number of bands. We estimate the positions of the guided bands *in gaps* of the unperturbed operator in terms of eigenvalues of Schrödinger operators on some finite graphs. We also determine sufficient conditions for the guided potentials under which the guided bands do not appear in gaps of the unperturbed problem.

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