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QUANTUM STRIPS IN HIGHER DIMENSIONS

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Abstract. We consider the Dirichlet Laplacian in unbounded strips on ruled surfaces in any space dimension. We locate the essential spectrum under the condition that the strip is asymptotically flat. If the Gauss curvature of the strip equals zero, we establish the existence of discrete spectrum under the condition that the curve along which the strip is built is not a geodesic. On the other hand, if it is a geodesic and the Gauss curvature is not identically equal to zero, we prove the existence of Hardy-type inequalities. We also derive an effective operator for thin strips, which enables one to replace the spectral problem for the Laplace-Beltrami operator on the two-dimensional surface by a one-dimensional Schrödinger operator whose potential is expressed in terms of curvatures.

In the appendix, we establish a purely geometric fact about the existence of relatively parallel adapted frames for any curve under minimal regularity hypotheses.

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REFERENCES

[1] D. BARSEGHYAN AND A. KHRABUSTOVSKYI, *Spectral estimates for Dirichlet Laplacian on tubes with exploding twisting velocity*, Oper. Matrices **13** (2019), 311–322.
[2] R. L. BISHOP, *There is more than one way to frame a curve*, The American Mathematical Monthly **82** (1975), 246–251.
[3] V. BRUNEAU, P. MIRANDA, D. PARRA, AND N. POPOFF, *Eigenvalue and resonance asymptotics in perturbed periodically twisted tubes: Twisting versus bending*, arXiv:1903.10599 (2019).
[4] V. BRUNEAU, P. MIRANDA, AND N. POPOFF, *Resonances near thresholds in slightly twisted waveguides*, Proc. Amer. Math. Soc. **146** (2018), 4801–4812.
[5] B. CHENAUD, P. DUCLOS, P. FREITAS, AND D. KREJČIŘÍK, *Geometrically induced discrete spectrum in curved tubes*, Differential Geom. Appl. **23** (2005), no. 2, 95–105.
[6] C. R. DE OLIVEIRA, L. HARTMANN, AND A. A. VERRI, *Effective Hamiltonians in surfaces of thin quantum waveguides*, J. Math. Phys. **60** (2019), 022101.
[7] C. R. DE OLIVEIRA AND A. F. ROSSINI, *Effective operators for Robin Laplacian in thin two- and three-dimensional curved waveguides*, preprint.
[8] C. R. DE OLIVEIRA AND A. A. VERRI, *Mild singular potentials as effective Laplacians in narrow strips*, Math. Scand. **120** (2017), 145–160.
[9] C. R. DE OLIVEIRA AND A. A. VERRI, *Norm resolvent approximation of thin homogeneous tubes by heterogeneous ones*, Commun. Contemp. Math. **19** (2017), 1650060.
[10] T. EKHOLM, H. KOVAŘÍK, AND D. KREJČIŘÍK, *A Hardy inequality in twisted waveguides*, Arch. Ration. Mech. Anal. **188** (2008), 245–264.
[11] P. EXNER AND P. ŠEBA, *Bound states in curved quantum waveguides*, J. Math. Phys. **30** (1989), 2574–2580.
[12] R. FERREIRA, L. M. MASCARENHAS, AND A. PIATNITSKI, *Spectral analysis in thin tubes with axial heterogeneities*, Portugal. Math. **72** (2015), 247–266.
[13] R. FROESE AND I. HERBST, *Realizing holonomic constraints in classical and quantum mechanics*, Commun. Math. Phys. **220** (2001), 489–535.

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[14] D. GILBARG AND N. S. TRUDINGER, *Elliptic partial differential equations of second order*, Springer-Verlag, Berlin, 1983.
[15] S. JIMBO AND K. KURATA, *Asymptotic behavior of eigenvalues of the Laplacian on a thin domain under the mixed boundary condition*, Indiana Univ. Math. J. **65** (2016), 867–898.
[16] T. KATO, *Perturbation theory for linear operators*, Springer-Verlag, Berlin, 1966.
[17] J. VON KELLER AND S. TEUFEL, *The NLS limit for bosons in a quantum waveguide*, Ann. H. Poincaré **17** (2016), 3321–3360.
[18] W. KLINGENBERG, *A course in differential geometry*, Springer-Verlag, New York, 1978.
[19] M. KOLB AND D. KREJČIŘÍK, *The Brownian traveller on manifolds*, J. Spectr. Theory **4** (2014), 235–281.
[20] D. KREJČIŘÍK, *Quantum strips on surfaces*, J. Geom. Phys. **45** (2003), no. 1–2, 203–217.
[21] D. KREJČIŘÍK, *Hardy inequalities in strips on ruled surfaces*, J. Inequal. Appl. **2006** (2006), Article ID 46409, 10 pages.
[22] D. KREJČIŘÍK, *Twisting versus bending in quantum waveguides*, Analysis on Graphs and its Applications, Cambridge, 2007 (P. Exner et al., ed.), Proc. Sympos. Pure Math., vol. 77, Amer. Math. Soc., Providence, RI, 2008, pp. 617–636. See arXiv:0712.3371v2 [math–ph] (2009) for a corrected version.
[23] D. KREJČIŘÍK, *Waveguides with asymptotically diverging twisting*, Appl. Math. Lett. **46** (2015), 7–10.
[24] D. KREJČIŘÍK AND R. TIEDRA DE ALDECOA, *Ruled strips with asymptotically diverging twisting*, Ann. H. Poincaré **19** (2018), 2069–2086.
[25] D. KREJČIŘÍK AND J. KŘÍŽ, *On the spectrum of curved quantum waveguides*, Publ. RIMS, Kyoto University **41** (2005), no. 3, 757–791.
[26] D. KREJČIŘÍK AND Z. LU, *Location of the essential spectrum in curved quantum layers*, J. Math. Phys. **55** (2014), 083520.
[27] D. KREJČIŘÍK, N. RAYMOND, J. ROYER, AND P. SIEGL, *Reduction of dimension as a consequence of norm-resolvent convergence and applications*, Mathematika **64** (2018), 406–429.
[28] D. KREJČIŘÍK AND H. ŠEDIVÁKOVÁ, *The effective Hamiltonian in curved quantum waveguides under mild regularity assumptions*, Rev. Math. Phys. **24** (2012), 1250018.
[29] J. LAMPART AND S. TEUFEL, *The adiabatic limit of Schrödinger operators on fibre bundles*, Math. Anal. **367** (2017), 1647–1683.
[30] C. R. MAMANI AND A. A. VERRI, *Absolute continuity and band gaps of the spectrum of the Dirichlet Laplacian in periodic waveguides*, Bull. Braz. Math. Soc. **49** (2018), 495–513.
[31] C. R. MAMANI AND A. A. VERRI, *Influence of bounded states in the Neumann Laplacian in a thin waveguide*, Rocky Mt. J. Math. **48** (2018), 1993–2021.
[32] F. MÉHATS AND N. RAYMOND, *Strong confinement limit for the nonlinear Schrödinger equation constrained on a curve*, Ann. H. Poincaré **18** (2017), 281–306.
[33] M. SPIVAK, *A comprehensive introduction to differential geometry*, vol. I, Publish or Perish, Houston, Texas, 2005.
[34] A. A. VERRI, *Dirichlet Laplacian in a thin twisted strip*, Int. J. Math. **30** (2019), 1950006.
[35] T. YACHIMURA, *Two-phase eigenvalue problem on thin domains with Neumann boundary condition*, Differ. Integral. Equ. **31** (2018), 735–760.
[36] A. ZETTL, *Sturm-Liouville theory*, Amer. Math. Soc., 2010.