

LOWER ESTIMATES ON EIGENVALUES OF QUANTUM GRAPHS

DELIO MUGNOLO AND MARVIN PLÜMER

Abstract. A method for estimating the spectral gap along with higher eigenvalues of quantum graphs has been introduced by Amini and Cohen-Steiner in [1] recently: it is based on a new transference principle between discrete and continuous models of a graph. We elaborate on it by developing a more general transference principle and by proposing alternative ways of applying it. To illustrate our findings, we present several spectral estimates on planar metric graphs that are oftentimes sharper than those obtained by isoperimetric inequalities and further previously known methods.

Mathematics subject classification (2010): 34B45 (05C50, 35P15, 81Q35).

Keywords and phrases: Spectral geometry of quantum graphs, planar graphs, double cover conjecture, normalized Laplacians.

REFERENCES

- [1] O. AMINI AND D. COHEN-STEINER, *A transfer principle and applications to eigenvalue estimates for graphs*, Comment. Math. Helv., 93:203–223, 2018.
- [2] S. ARITURK, *Eigenvalue estimates on quantum graphs*, arXiv:1609.07471, 2016.
- [3] D. BORTHWICK, L. CORSI AND K. JONES, *Sharp diameter bound on the spectral gap for quantum graphs*, arXiv:1905.03071, 2019.
- [4] J. VON BELOW, *A characteristic equation associated with an eigenvalue problem on c^2 -networks*, Lin. Algebra Appl., 71:309–325, 1985.
- [5] G. BERKOLAIKO AND P. KUCHMENT, *Introduction to Quantum Graphs*, volume 186 of *Math. Surveys and Monographs*, Amer. Math. Soc., Providence, RI, 2013.
- [6] G. BERKOLAIKO, J.B. KENNEDY, P. KURASOV AND D. MUGNOLO, *Edge connectivity and the spectral gap of combinatorial and quantum graphs*, J. Phys. A, 50:365201, 2017.
- [7] G. BERKOLAIKO, J.B. KENNEDY, P. KURASOV AND D. MUGNOLO, *Surgery principles for the spectral analysis of quantum graphs*, Trans. Amer. Math. Soc., (to appear).
- [8] F. BAUER, M. KELLER, AND R. WOJCIECHOWSKI, *Cheeger inequalities for unbounded graph Laplacians*, J. European Math. Soc., 17:259–271, 2015.
- [9] R. BAND AND G. LÉVY, *Quantum graphs which optimize the spectral gap*, Ann. Henri Poincaré, 18:3269–3323, 2017.
- [10] F.R.K. CHUNG, *Spectral Graph Theory*, volume 92 of *Reg. Conf. Series Math.*, Amer. Math. Soc., Providence, RI, 1997.
- [11] L. FRIEDLANDER, *Extremal properties of eigenvalues for a metric graph*, Ann. Inst. Fourier, 55:199–212, 2005.
- [12] D. HORAK AND J. JOST, *Spectra of combinatorial Laplace operators on simplicial complexes*, Advances in Math., 244:303–336, 2013.
- [13] F. JAEGER, *A survey of the cycle double cover conjecture*, In B.R. Alspach and C.D. Godsil, editors, *Annals of Discrete Mathematics: Cycles in Graphs*, volume 115 of *North-Holland Math. Studies*, pages 1–12. North-Holland, Amsterdam, 1985.
- [14] J.B. KENNEDY, *A family of diameter-based eigenvalue bounds for quantum graphs*, arXiv:1807.08185, 2018.
- [15] J.B. KENNEDY, P. KURASOV, G. MALENOVÁ AND D. MUGNOLO, *On the spectral gap of a quantum graph*, Ann. Henri Poincaré, 17:2439–2473, 2016.

- [16] M. KRAMAR FIJAVŽ, D. MUGNOLO AND E. SIKOLYA, *Variational and semigroup methods for waves and diffusion in networks*, Appl. Math. Optim., 55:219–240, 2007.
- [17] P. KURASOV AND S. NABOKO, *Rayleigh estimates for differential operators on graphs*, J. Spectral Theory, 4:211–219, 2014.
- [18] A. KOSTENKO AND N. NICOLUSSI, *Spectral estimates for infinite quantum graphs*, Calc. Var., 58:15, 2019.
- [19] D. LENZ, M. SCHMIDT AND P. STOLLMANN, *Topological Poincaré type inequalities and lower bounds on the infimum of the spectrum for graphs*, arXiv:1801.09279, 2018.
- [20] D.S. MITRINOVIC, *Analytic Inequalities, volume 165 of Grundlehren der mathematischen Wissenschaften*, Springer-Verlag, Berlin, 1970.
- [21] D. MUGNOLO, *Semigroup Methods for Evolution Equations on Networks*, Underst. Compl. Syst. Springer-Verlag, Berlin, 2014.
- [22] S. NICAISE, *Spectre des réseaux topologiques finis*, Bull. Sci. Math., II. Sér., 111:401–413, 1987.
- [23] J. ROHLEDER, *Eigenvalue estimates for the Laplacian on a metric tree*, Proc. Amer. Math. Soc., 145:2119–2129, 2016.
- [24] K. RUEDENBERG AND C.W. SCHERR, *Free-electron network model for conjugated systems. I. Theory*, J. Chem. Phys., 21:1565–1581, 1953.
- [25] M. SOLOMYAK, *On eigenvalue estimates for the weighted laplacian on metric graphs*, In M.Sh. Birman, S. Hildebrandt, V.A. Solonnikov, and N.N. Uraltseva, editors, *Nonlinear Problems in Mathematical Physics and Related Topics I*, pages 327–347. Springer-Verlag, New York, 2002.
- [26] M.J. WENNINGER, *Dual models*, Cambridge Univ. Press, 2003.
- [27] H. WHITNEY, *Congruent Graphs and the Connectivity of Graphs*, Amer. Jour. of Math., 54(1):150–168, 1932.
- [28] S. WILSON, *Operators over regular maps*, Pacific J. Math., 81:559–568, 1979.
- [29] C.-Q. ZHANG, *Circuit Double Cover of Graphs, volume 399 of London Math. Soc. Lect. Note Series*, Cambridge University Press, Cambridge, 2012.