AN EXTENSION OF THE BEURLING–CHEN–HADWIN–SHEN THEOREM FOR NONCOMMUTATIVE HARDY SPACES ASSOCIATED WITH FINITE VON NEUMANN ALGEBRAS

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Abstract. In 2015, Yanni Chen, Don Hadwin and Junhao Shen proved a noncommutative version of Beurling’s theorems for a continuous unitarily invariant norm $\alpha$ on a tracial von Neumann algebra $(\mathcal{M}, \tau)$, where $\alpha$ is $\|\cdot\|_1$-dominating with respect to $\tau$. In the paper, we first define a class of norms $N_\Delta(\mathcal{M}, \tau)$ on $\mathcal{M}$, called determinant, normalized, unitarily invariant continuous norms on $\mathcal{M}$. If $\alpha \in N_\Delta(\mathcal{M}, \tau)$, then there exists a faithful normal tracial state $\rho$ on $\mathcal{M}$ such that $\rho(x) = \tau(xg)$ for some positive $g \in L^1(\mathcal{Z}, \tau)$ and the determinant of $g$ is positive. For every $\alpha \in N_\Delta(\mathcal{M}, \tau)$, we study the noncommutative Hardy spaces $H^\alpha(\mathcal{M}, \tau)$, then prove that the Chen-Hadwin-Shen theorem holds for $L^\alpha(\mathcal{M}, \rho)$. The key ingredients in the proof of our result include a factorization theorem and a density theorem for $L^\alpha(\mathcal{M}, \rho)$.


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REFERENCES