

HARMONIC HARDY SPACE AND THEIR OPERATORS

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Abstract. Let H^2 be the Hardy space on the unit disk. For inner functions u and v , the harmonic Hardy space $H_{u,v}^2$ is defined by $H_{u,v}^2 = uH^2 \oplus v\bar{z}H^2$. Assume one of u and v is a nonconstant, then $H_{u,v}^2$ is a proper closed subspace of $L^2(\partial\mathbb{D})$. We can define the Toeplitz operator on the $H_{u,v}^2$ by $\hat{T}_f x = Qfx$ for $x \in H_{u,v}^2$, where Q is the orthogonal projection from $L^2(\partial\mathbb{D})$ onto $H_{u,v}^2$. We studied some algebraic properties of the Toeplitz operator on $H_{u,v}^2$ and obtained some interesting results that are different from the Toeplitz operators in the classical function space.

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