A NOTE ON BAPAT'S q–PERMANENT CONJECTURE

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Abstract. Ravindra Bapat conjectured the q-permanent of a non-diagonal Hermitian positive definite matrix is a strictly increasing (in q) interpolation between the determinant ($q = -1$) and the permanent ($q = 1$). We prove that this is true for non-diagonal positive definite matrices if and only if it is true for singular positive semidefinite matrices without a zero row. Thus we conjecture the q-permanent of a non-diagonal Hermitian positive semidefinite matrix without a zero row is strictly increasing on $[-1,1]$. We prove this extended conjecture in the rank-one case and the 3-by-3 case.

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