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OPERATOR REPRESENTATIONS OF K–FRAMES: BOUNDEDNESS AND STABILITY

MIAO HE, JINSONG LENG, DONGWEI LI AND YUXIANG XU

Abstract. In this paper, a K-frame $\{f_k\}_{k\in Z}$ for a Hilbert space H, with the form $\{T^k f_0\}_{k\in Z}$ for an operator T is analyzed. Some conditions under which a K-frame can be represented by an operator and then investigate the properties of this operator are discussed. More Specifically, a necessary and sufficient condition for a K-frame that has an operator representation can be obtained by a K-dual. Furthermore, we find the boundedness of the operator T has an integral relationship with the operator K when a K-frame can be represented by an operator T. In addition, the stability of operator representation is studied. We prove that the stability and boundedness are preserved under certain restrictions on the perturbation condition. A pretty small perturbation will heavily affect the property of being representable by an operator if $\mu > 0$, and an example is used to illustrate it. Furthermore, some elements from a subspace of H are used to perturb a K-frame, and then some useful stability results are obtained.

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