

OPERATOR PARAMETERIZATIONS OF FRAME GENERATORS AND GENERALIZED DUAL PAIR OF FRAME GENERATORS OF UNITARY SYSTEMS

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Abstract. In this paper, we first give a condition such that a countable unitary system has complete wandering vectors, which plays the key role to operator parameterize frame generators for unitary systems in the literatures. Then we continue to study more general operator parameterizations and their applications. Based on the invertibility of some parameterizing operators, we introduce the concept of generalized dual pair of frame generators for a unitary system, which is a natural generalization of dual pair of frame generators. Then we characterize generalized dual pair of frame generators and some interesting properties and constructions of generalized dual frame generator pairs are also studied.

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