EXPLICIT KREIN RESOLVENT IDENTITIES FOR
SINGULAR STURM—LIOUVILLE OPERATORS
WITH APPLICATIONS TO BESSEL OPERATORS

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Abstract. We derive explicit Krein resolvent identities for generally singular Sturm–Liouville
operators in terms of boundary condition bases and the Lagrange bracket. As an application
of the resolvent identities obtained, we compute the trace of the resolvent difference of a pair
of self-adjoint realizations of the Bessel expression $-\frac{d^2}{dx^2} + (\nu^2 - (1/4))x^{-2}$ on $(0, \infty)$ for
values of the parameter $\nu \in [0, 1)$ and use the resulting trace formula to explicitly determine the
spectral shift function for the pair.

Keywords and phrases: Krein identity, singular Sturm–Liouville operator, Bessel operator, spectral
shift function.

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