Operators and Matrices
Volume 14, Number 4 (2020), 1101–1125
doi:10.7153/oam-2020-14-66

RANDOM STURM–LIOUVILLE OPERATORS WITH GENERALIZED POINT INTERACTIONS

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Abstract. In this work we study the point spectra of selfadjoint Sturm-Liouville operators with generalized point interactions, where the two one-sided limits of the solution data are related via a general $\mathrm{SL}(2,\mathbb{R})$ matrix. We are particularly interested in the stability of eigenvalues with respect to the variation of the parameters of the interaction matrix. As a particular application to the case of random generalized point interactions we establish a version of Pastur's theorem, stating that except for degenerate cases, any given energy is an eigenvalue only with probability zero. For this result, independence is important but identical distribution is not required, and hence our result extends Pastur's theorem from the ergodic setting to the non-ergodic setting.

Mathematics subject classification (2010): 34L05, 47E05, 47N99.
Keywords and phrases: Sturm-Liouville operators, point interactions, eigenvalue problem.

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