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THE NORM OF AN INFINITE L–MATRIX

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Abstract. Evaluating the norm of infinite matrices, as operators acting on the sequence space ℓ^2, is not an easy task. For a few celebrated matrices, e.g., the Hilbert matrix and the Cesàro matrix, the precise value of the norm is known. But, for many other important cases we use estimated values of norm. In this note, we study the norm of L-matrices $A=[a_n]$, which appear in studying Hadamard multipliers of function spaces. We provide some necessary and sufficient conditions for the finiteness of norm and study the sharpness of these conditions. In particular, for the decay rate $a_n=O(1/n^\alpha)$, our characterization is complete. Finally, parallel to the above classical results of Hilbert and Cesàro, we succeed to show that $\|A_s\|=4$ for the family of L-matrices $A_s=[1/(n+s)]$, irrelevant of the parameter s which runs over $[1/2,\infty)$.

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Paper OaM-15-04

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