

NEW DETERMINANTAL INEQUALITIES CONCERNING HERMITIAN AND POSITIVE SEMI-DEFINITE MATRICES

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Abstract. Let A, B be $n \times n$ matrices such that A is positive semi-definite and B is Hermitian. In this note, it is shown, among other inequalities, the following determinantal inequality

$$\det(A^k + (AB)^2) \geq \det(A^k + A^2B^2)$$

for all $k \in [1, \infty[$.

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REFERENCES

- [1] H. ABBAS AND M. GHABRIES, *Some generalizations and complements of determinantal inequalities*, Math. Inequal. Appl. **23**, 1 (2020), 169–176.
- [2] K. M. R. AUDENAERT, *A determinantal inequality for the geometric mean with an application in diffusion tensor imaging*, arXiv **2**, 1502.06902 (2015).
- [3] R. BHATIA, *Matrix Analysis*, Grad. Texts in Math., vol. 169, Springer-Verlag, New York, 1997.
- [4] R. BHATIA, *Positive Definite Matrices*, Princeton University Press, New Jersey, 2007.
- [5] F. KITTANEH, M. LIN, *Trace inequalities for positive semidefinite block matrices*, Linear Algebra and its Applications **524**, (2017), 153–156.
- [6] M. LIN, *On a determinantal inequality arising from diffusion tensor imaging*, Commun. Contemp. Math **19**, 1650044 (2017), 6 pp.
- [7] M. LIN, H. WOLKOWICZ, *An eigenvalue majorization inequality for positive semi-definite block matrices*, Linear Multilinear Algebra **60**, (2011), 1365–1368.
- [8] L. PLEVNIK, *On a matrix trace inequality due to Ando, Hiai and Okubo*, Indian J. Pure Appl. Math **47** (3), (2015), 491–500.
- [9] M. SAL MOSLEHIAN, MOHSEN KIAN, AND QINGXIANG XU, *Positivity of 2×2 block matrices of operators*, Banach J. Math. Anal. **13** 3, (2019), 726–743.
- [10] F. ZHANG, *Matrix Theory: Basic Results and Techniques*, Springer, 2nd ed., New York, 2011.