COMPLEX SYMMETRY OF WEIGHTED COMPOSITION OPERATORS ON A HILBERT SPACE OF DIRICHLET SERIES

XINGXING YAO

Abstract. In this paper, some characterizations of symbols ϕ and φ are provided when the weighted composition operators $C_{\phi,\varphi}$ are complex symmetric on the Hilbert space of Dirichlet series with square summable coefficients. As an application, it is proved that there are non-trivial complex symmetric and non-normal weighted composition operators, although this phenomenon happens for neither composition operators nor multiplication operators.

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