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COMMUTING AND SEMI–COMMUTING TOEPLITZ OPERATORS ON THE WEIGHTED HARMONIC BERGMAN SPACE

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Abstract. In this article, we show that two Toeplitz operators on the weighted harmonic Bergman space can commute only in the trivial case under certain conditions. The triviality here means a nonzero linear combination of their symbols is constant. Moreover, we give a characterization of semi-commuting Toeplitz operators with harmonic or analytic symbols.
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