

THE REFLEXIVITY OF HYPEREXPANSIONS AND THEIR CAUCHY DUAL OPERATORS

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Abstract. We discuss the reflexivity of hyperexpansions and their Cauchy dual operators. In particular, we show that any cyclic completely hyperexpansive operator is reflexive. We also establish the reflexivity of the Cauchy dual of an arbitrary 2-hyperexpansive operator. As a consequence, we deduce the reflexivity of the so-called Bergman-type operator, that is, a left-invertible operator T satisfying the inequality $TT^* + (T^*T)^{-1} \leq 2I_{\mathcal{H}}$.

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