ON THE DRAZIN INVERSE AND M–P
INVERSE FOR SUM OF MATRICES

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Abstract. Drazin inverse and M-P inverse have many important applications in the aspects of theoretic research of operator and statistics. In this article, we will exhibit under suitable conditions a neat relationship between the Drazin inverse of $A + B$ and the Drazin inverses of the individual terms A and B. Furthermore, with the same thread, we will give an expression of the M-P inverse of $A + B$ in terms of only the M-P inverses of matrices A and B.

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