

REFINED HEINZ OPERATOR INEQUALITIES AND NORM INEQUALITIES

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Abstract. In this article we study the Heinz and Hermite-Hadamard inequalities. We derive the whole series of refinements of these inequalities involving unitarily invariant norms, which improve some recent results, known from the literature.

We also prove that if $A, B, X \in M_n(\mathbb{C})$ such that A and B are positive definite and f is an operator monotone function on $(0, \infty)$. Then

$$\|f(A)X - Xf(B)\| \leq \max\{\|f'(A)\|, \|f'(B)\|\} \|AX - XB\|.$$

Finally we obtain a series of refinements of the Heinz operator inequalities, which were proved by Kittaneh and Krnić.

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