

ON THE COMMUTATOR AND FREDHOLMNESS OF ISOMETRIC PAIR

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Abstract. In this paper, we characterize the Fredholmness and compactness of commutators of the commuting isometric pair $W = (W_0, W_1)$ by using of their canonical model [3].

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