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THE PRODUCT OF OPERATORS AND THEIR THE MOORE–PENROSE INVERSES ON HILBERT C^*–MODULES

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Abstract. We assure the existence of the Moore–Penrose inverse of a product UTS, under the assumptions that T has a closed range and that there exist U' and S' such that $U'UT = T = TSS'$, and then we characterize the Moore–Penrose inverse of UTS in terms of the corresponding inverses of T. Also, we obtain the block matrix decomposition of operators, which implies that the reverse order law for operators establishes. Finally we achieve some relations between the product of operators and their the Moore-Penrose inverses.

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