THE CONVEX INVERTIBLE CONE STRUCTURE OF
POSITIVE REAL ODD RATIONAL MATRIX FUNCTIONS

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Abstract. Positive real odd matrix functions, often referred to as positive real lossless matrix functions, play an important role in many applications in multi-port electrical systems. In this paper we present closer analogues to some of the known results for the scalar, one-port, case in the multi-port setting. Specifically, we determine necessary and sufficient conditions for the well studied partial fraction formula to represent functions in the class of positive real odd matrix functions, and explicit minimal state space realization formulas for the inverse (admittance) of a function in this class, which itself is also a positive real odd matrix function. Doing so, enables us to provide a partial analogue of the pole-zero interlacing behavior from the scalar case.

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