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NUMERICAL RANGES ENCIRCLED BY ANALYTIC CURVES

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Abstract. Let D be a bounded convex domain in $\mathbb{C}$ with a regular analytic boundary. Suppose that the numerical range $W(A)$ of a bounded linear operator A is contained in $\overline{D}$. If $W(A)$ intersects the boundary ∂D at infinitely many points while the essential numerical range $W_{\text{ess}}(A)$ does not intersect ∂D, then $W(A) = \overline{D}$. This generalizes some infinite dimensional analogues of a result of Anderson [2, 4].

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