

CLOSEDNESS OF RANGES OF UNBOUNDED UPPER TRIANGULAR OPERATOR MATRICES

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Abstract. This paper deals with the closed range property of operator matrices. The necessary and sufficient condition is given for an unbounded upper triangular partial operator matrix to have a closed range completion. In particular, the bounded case is its direct consequence.

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