

## SHARP OPERATOR MEAN INEQUALITIES OF THE NUMERICAL RADII

HOSNA JAFARMANESH AND MARYAM KHOSRAVI

**Abstract.** We present several sharp upper bounds and some extension for product operators. Among other inequalities, it is shown that if  $0 < mI \leq B^* f^2(|X|)B$ ,  $A^* g^2(|X^*|)A \leq MI$ ,  $f, g$  are non-negative continuous functions on  $[0, \infty)$  such that  $f(t)g(t) = t$ , ( $t \geq 0$ ), then for all non-negative operator monotone decreasing function  $h$  on  $[0, \infty)$ , we obtain that

$$\|h(B^* f^2(|X|)B) \sigma h(A^* g^2(|X^*|)A)\| \leq \frac{mk}{M} h(\langle (A^*XB)x, x \rangle),$$

As an application of the above inequality, it is shown that

$$\omega(A^*XB) \leq \frac{mk}{M} \|B^* f^2(|X|)B A^* g^2(|X^*|)A\|,$$

where,  $k = \frac{(M+m)^2}{4mM}$  and  $\sigma$  is an operator mean s.t.,  $! \leq \sigma \leq \nabla$ .

**Mathematics subject classification (2020):** Primary 47A63; Secondary 47A64.

**Keywords and phrases:** Numerical radius, operator norm, inequality, refine.

### REFERENCES

- [1] J. S. AUJLA AND F. C. SILVA, *Weak majorization inequalities and convex functions*, Linear Algebra Appl., 369 (2003), 217–233.
- [2] T. ANDO AND F. HIAI, *Operator log-convex functions and operator means*, Math. Ann. 350 (2011), 611–630
- [3] A. ABU-OMAR AND F. KITTANEH, *A numerical radius inequality involving the generalized Aluthge transform*, Studia Math. 216 (2013), 69–75.
- [4] R. BHATIA, *Matrix Analysis*, Springer, New York (1997).
- [5] S. S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Sarajevo J. Math. 5 (18) (2) (2009) 269–278.
- [6] H. JAFARMANESH, M. KHOSRAVI AND A. SHEIKHHOSSEINI, *Some operator inequalities involving operator monotone functions*, Bulletin des Sciences Mathematiques, 2021, doi:10.1016/j.bulsci.2020.102938.
- [7] F. KITTANEH, *Notes on some inequalities for Hilbert space operators*, Publ. Res. Ints. Math. Sci., 1988, 24 (2): 283–293.
- [8] F. KITTANEH, *Norm inequalities for sums of positive operators*, J. Operator Theory 48 (2002), 95–103.
- [9] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math. 168 1 (2005), 73–80.
- [10] B. MOND AND J. PECARIC, *On Jensen's inequality for operator convex functions*, Houston J Math, 1995, 21: 739–753.
- [11] K. SHEBRAWI AND H. ALBADAWI, *Numerical radius and operator norm inequalities*, J. Inequal. Appl. 2009, Art. ID 492154, 11-pp.
- [12] M. SATTARI, M. S. MOSLEHIAN AND T. YAMAZAKI, *Some generalized numerical radius inequalities for Hilbert space operators*, Linear Algebra Appl. 470 (2014), 1–12.
- [13] T. YAMAZAKI, *On upper and lower bounds of the numerical radius and on equality condition*, Studia Math. 178, (2007), 83–89.