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A BISHOP–PHELPS–BOLLOBÁS TYPE PROPERTY FOR MINIMUM ATTAINING OPERATORS

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Abstract. In this article, we study the Bishop-Phelps-Bollobás type theorem for minimum attaining operators. More explicitly, if we consider a bounded linear operator T on a Hilbert space H and a unit vector $x_0 \in H$ such that $\|Tx_0\|$ is very close to the minimum modulus of T, then T and x_0 are simultaneously approximated by a minimum attaining operator S on H and a unit vector $y \in H$ for which $\|Sy\|$ is equal to the minimum modulus of S. Further, we extend this result to a more general class of densely defined closed operators (need not be bounded) in Hilbert space. As a consequence, we get the denseness of the set of minimum attaining operators in the class of densely defined closed operators with respect to the gap metric.

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