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INEQUALITIES RELATED TO THE GEOMETRIC MEAN OF ACCRETIVE MATRICES

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Abstract. We present some inequalities related to the recently defined geometric mean of two accretive matrices. Firstly, we show that if the block matrix $\begin{pmatrix} A & X \\ Y^* & B \end{pmatrix}$ is accretive, then the singular values of $(X+Y)/2$ are weakly log majorized by the singular values of the geometric mean of A and B. This extends a result of M. Lin.

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