PROPERTIES OF J−SELF−ADJOINT OPERATORS

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Abstract. In this paper, we consider operators $T \in \mathscr{L}(\mathscr{H})$ such that $(JT)^* = JT$ for some anti-unitary J with $J^2 = -I$; in this case, we say that T is J-self-adjoint. We show that the Aluthge transform of a J-self-adjoint operator is skew-complex symmetric. As an application, we prove that w-hyponormal operators which are J-self-adjoint must be normal. Moreover, we obtain that if $T \in \mathscr{L}(\mathscr{H})$ is a J-self-adjoint operator with property (β), then $T + A$ is decomposable where $A \in \mathscr{L}(\mathscr{H})$ is an algebraic operator commuting with T. We also give examples of J-self-adjoint operators.

Mathematics subject classification (2020): 47A11, 47A10, 47B49, 47B32.
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