

## RATIONAL ELIMINATION ALGORITHM AND APPLICATIONS

BÉCHIR DALI AND MOHAMED ELGHAOUI

*Abstract.* Given a matrix  $A \in \mathbb{R}^{m \times n}$ , we develop an algorithm called the rational elimination algorithm which resembles the algorithm elimination except that the pivot (the leading coefficient) is a sequence of real independent numbers over  $\mathbb{Q}$ . This algorithm is used to calculate the rank of  $A$  over  $\mathbb{Q}$  and to seek rational solutions to any linear system  $Ax = b$  with  $b \in \mathbb{R}^m$ . We also present a criterion for testing the density of additive subgroups of  $\mathbb{R}^n$  which needs the rank of a certain matrix over  $\mathbb{Q}$ . Finally, we apply such algorithm for testing the regularity of an orbit under the linear continuous action of some subgroup of  $GL(V)$  where  $V$  is finite dimensional real vector space.

*Mathematics subject classification (2020):* 15A06, 15A24, 15B36, 11J72.

*Keywords and phrases:* Irrationality, linear dependence over a field, rational numbers, rational elimination algorithm.

### REFERENCES

- [1] A. AYADI AND H. MARZOUGUI, *Dense orbits for abelian subgroups of  $GL(n, \mathbb{C})$* , *Foliations 2005*: World Scientific, Hackensack, NJ, (2006), 47–69.
- [2] D. ARNAL, B. DALI, B. CURREY, V. OUSSA, *Regularity of abelian linear actions*, *Commutative and noncommutative harmonic analysis and applications*, 89–109, *Contemp. Math.*, 603, Amer. Math. Soc., Providence, RI, 2013.
- [3] DIDIER ARNAL, BRADLEY CURREY, VIGNON OUSSA, *Characterization of regularity for a connected Abelian action*, *Monatsh. Math.* Volume 180. Issue 1, pp. 1–37, doi:10.1007/s00605-015-0811-y.
- [4] ELGHAOUI, MOHAMED, AND ADLENE AYADI, *Rational Criterion Testing the Density of Additive Subgroups of  $\mathbb{R}^n$  and  $\mathbb{C}^n$* , *Applied General Topology*, vol. 16, no. 2, Oct. 2015, pp. 127–139, doi:10.4995/agt.2015.3257.
- [5] MOHAMED ELGHAOUI AND ADLENE AYADI, *Algorithm testing for the hypercyclicity of finitely abelian subgroups of  $GL(n, \mathbb{C})$* , *Operators and Matrices* Volume 10, Number 3 (2016), 669–678, doi:10.7153/oam-10-40.
- [6] O. L. MANGASARIAN, AND M. C. FERRIS, *Uniqueness of Integer Solution of Linear Equations*, *Optimization Letters*, vol. 4, no. 4, Nov. 2010, pp. 559–65, doi:10.1007/s11590-010-0183-0.
- [7] M. WALDSCHMIDT, *Topologie des points rationnels*, *Cours de troisième Cycle*, Université P. et M. Curie (Paris VI), (1994/95).
- [8] MARTIN, RICHARD KIPP, *Large Scale Linear and Integer Optimization: A Unified Approach*, Springer US, 1999, doi:10.1007/978-1-4615-4975-8.