

APPLYING SOLVABILITY THEOREMS FOR MATRIX EQUATIONS

Z. MOUSAVI AND F. MIRZAPOUR

Abstract. In this paper, using solvability theorems for matrix equations, generally applicable results are proved for the existence of positive semidefinite or asymptotically positive semidefinite solution. In the following, a question about the matrix equation $f(A)X + Xf(A) = AB + BA$ is answered. This question was asked, first by Chan and Kwong [6] and then by Furuta [7].

Mathematics subject classification (2020): 15A24.

Keywords and phrases: Matrix equation, solvability theorem, positive definite solution, Farkas lemma.

REFERENCES

- [1] A. BEN-ISRAEL, *Linear Equations and Inequalities on Finite Dimensional, Real or Complex, Vector Spaces: A Unified Theory*, J. Math. Anal. Appl. 27, 367–389 (1969).
- [2] E. V. BELMEGA, S. LASAULCE AND M. DEBBAH, *A Trace inequality for positive definite matrices*, Journal of inequalities in pure and applied mathematics, Vol. 10 (2009), Issue 1, Article 5, 4 pp.
- [3] A. BERMAN AND A. BEN-ISRAEL, *More on linear inequalities with applications to matrix theory*, J. Math. Anal. Appl. 33 (1971), 482–496.
- [4] A. BERMAN AND A. BEN-ISRAEL, *Linear Equations over Cones with Interior: A Solvability Theorem with Applications to Matrix Theory*, Linear Algebra Appl. 7 (1973) 139–149.
- [5] R. BHATIA, *Matrix Analysis*, Grad. Texts in Math. 169, Springer-Verlag, New York, 1997.
- [6] N. N. CHAN AND M. K. KWONG, *Hermitian matrix inequalities and a conjecture*, Amer. Math. Monthly 92 (1985), 533–541.
- [7] T. FURUTA, *Positive semidefinite solutions of the operator equation $\sum_{j=1}^n A_{n-j}XA_{j-1} = B$* , Linear Algebra Appl. 432 (2010) 949–955.
- [8] P. LANCASTER AND L. RODMAN, *Algebraic Riccati Equations*, Oxford University Press, New York, 1995.
- [9] T. MORI, *Comments on “A Matrix Inequality Associated with Bounds on Solutions of Algebraic Riccati and Lyapunov Equation”*, IEEE Trans. Automat. Contr., vol. AC-33, p. 1088, Nov. 1988.
- [10] A. M. RAN AND M. C. B. REURINGS, *The symmetric linear matrix equation*, The Electronic Journal of Linear Algebra 9 (2002) 93–107.