

## SOME REMARKS IN $C^*$ - AND $K$ -THEORY

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*Abstract.* This note consists of three unrelated remarks. First, we demonstrate how roughly speaking  $*$ -homomorphisms between matrix stable  $C^*$ -algebras are exactly the uniformly continuous  $*$ -preserving group homomorphisms between their general linear groups. Second, using the Cuntz picture in  $KK$ -theory we bring morphisms in  $KK$ -theory represented by generators and relations to a particular simple form. Third, we show that for an inverse semigroup its associated groupoid is Hausdorff if and only if the inverse semigroup is  $E$ -continuous.

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### REFERENCES

- [1] B. BURGSTALLER, *A note on a certain Baum–Connes map for inverse semigroups*, Houston J. Math., Houston J. Math., 46 (3): 747–769, 2020.
- [2] B. BURGSTALLER, *The generators and relations picture of  $KK$ -theory*, preprint, arXiv:1602.03034v2.
- [3] B. BURGSTALLER, *The universal property of inverse semigroup equivariant  $KK$ -theory*, Kyungpook Math. J., 61 (1): 111–137, 2021.
- [4] B. BURGSTALLER, *Attempts to define a Baum–Connes map via localization of categories for inverse semigroups*, Aust. J. Math. Anal. Appl. Vol. 17 (2020), No. 2, Art. 1, 22 pp.
- [5] B. BURGSTALLER, *Inverse semigroup equivariant  $KK$ -theory and  $C^*$ -extensions*, Oper. Matrices, 10 (2): 467–484, 2016.
- [6] B. BURGSTALLER, *Semigroup homomorphisms on matrix algebras*, Adv. Operat. Th., 2 (3): 287–292, 2017.
- [7] J. CUNTZ,  *$K$ -theory and  $C^*$ -algebras*, Algebraic K-theory, number theory, geometry and analysis, Proc. int. Conf., Bielefeld/Ger. 1982., Lect. Notes Math. 1046, 55–79 (1984), 1984.
- [8] J. CUNTZ, *A new look at  $KK$ -theory*,  $K$ -Theory, 1 (1): 31–51, 1987.
- [9] R. EXEL AND E. PARDO, *The tight groupoid of an inverse semigroup*, Semigroup Forum, 92: 274–303, 2016.
- [10] N. HIGSON, *A characterization of  $KK$ -theory*, Pac. J. Math., 126 (2): 253–276, 1987.
- [11] G. G. KASPAROV, *Equivariant  $KK$ -theory and the Novikov conjecture*, Invent. Math., 91 (1): 147–201, 1988.
- [12] M. KHOSHKAM AND G. SKANDALIS, *Regular representation of groupoid  $C^*$ -algebras and applications to inverse semigroups*, J. Reine Angew. Math., 546: 47–72, 2002.
- [13] R. MEYER AND R. NEST, *The Baum–Connes conjecture via localisation of categories*, Topology, 45 (2): 209–259, 2006.
- [14] L. MOLNÁR,  *$*$ -semigroup endomorphisms of  $B(H)$* , In Recent advances in operator theory and related topics. The Béla Szőkefalvi-Nagy memorial volume. Proceedings of the memorial conference, Szeged, Hungary, August 2–6, 1999, pages 465–472, Basel: Birkhäuser, 2001.
- [15] A. L. T. PATERSON, *Groupoids, inverse semigroups, and their operator algebras*, volume 170, Boston, MA: Birkhäuser, 1999.
- [16] B. STEINBERG, *A groupoid approach to discrete inverse semigroup algebras*, Adv. Math., 223: 689–727, 2010.

- [17] B. STEINBERG, *Simplicity, primitivity and semiprimitivity of étale groupoid algebras with applications to inverse semigroup algebras*, J. Pure Appl. Algebra, 220 (3): 1035–1054, 2016.
- [18] K. THOMSEN, *The universal property of equivariant  $KK$ -theory*, J. Reine Angew. Math., 504: 55–71, 1998.
- [19] J. L. TU, *Non-Hausdorff groupoids, proper actions and  $K$ -theory*, Doc. Math., 9: 565–597, 2004.