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RECOGNITION OF MATRICES WHICH ARE SIGN–REGULAR OF A GIVEN ORDER AND A GENERALIZATION OF OSCILLATORY MATRICES

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Abstract. In this paper, rectangular matrices whose minors of a given order have the same strict sign are considered and sufficient conditions for their recognition are presented. The results are extended to matrices whose minors of a given order have the same sign or are allowed to vanish. A matrix A is called oscillatory if all its minors are nonnegative and there exists a positive integer k such that A^k has all its minors positive. As a generalization, a new type of matrices, called oscillatory of a specific order, is introduced and some of their properties are investigated.

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