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# CHARACTERIZATIONS AND REPRESENTATIONS FOR THE DRAZIN INVERSE OF ANTI–TRIANGULAR BLOCK OPERATOR MATRICES WITH INDEX LESS THAN TWO

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*Abstract.* In the 1980s, Campbell proposed a problem to find an expression of the Drazin inverse for the block matrix $\begin{pmatrix} A & B \\ -I & 0 \end{pmatrix}$ to research on singular differential equations. In this paper, some characterizations and detail representations for the Drazin inverse of anti-triangular block operator matrices $M = \begin{pmatrix} A & B \\ I & 0 \end{pmatrix}$ with $ind(M) = 1$ ( resp. $ind(M) = 2$ ) are given.

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## REFERENCES

[1] M. BARRAA, E. H. BENABDI, *On equivalence of linear combinations of idempotents*, Linear Multilinear Algebra 152 (2018) 38–64.
[2] A. BEN-ISRAL, T. N. E. GREVILLE, *Generalized Inverses: Theory and Applications*, 2nd ed, Springer-Verlag, 2003.
[3] C. J. BU, K. Z. ZHANG AND J. M. ZHAO, *Representations of the Drazin inverse on solution of a class singular differential equations*, Linear Multilinear Algebra 8 (2011) 863–877.
[4] S. L. CAMPBELL, *The Drazin inverse and systems of second order linear differential equations*, Linear Multilinear Algebra 14 (1983) 195–198.
[5] S. L. CAMPBELL AND C. D. MEYER, *Generalized Inverse of Linear Transformations*, London, Pitman, 1979.
[6] S. L. CAMPBELL, *Singular systems of differential equations*, London, Pitman, 1980.
[7] S. L. CAMPBELL, C. D. MEYER JR AND N. J. ROSE, *Applications of the Drazin inverse to linear systems of differential equations with singular constant coefficients*, SIAM J. Appl. Math. 31 (1976) 411–425.
[8] N. CASTRO-GONZÁLEZ, *Additive perturbation results for the Drazin inverse*, Linear Algebra Appl. 397 (2005) 279–297.
[9] N. CASTRO-GONZÁLEZ, E. DOPAZO, *Representations of the Drazin inverse for a class of block matrices*, Linear Algebra Appl. 400 (2005) 253–269.
[10] N. CASTRO-GONZÁLEZ, J. J. KOLIHA, *New additive results for the g-Drazin inverse*, Proc. Roy. Soc. Edinburgh Sect. A 134 (2004) 1085–1097.
[11] J. J. CLIMENT, M. NEUMANN, AND A. SIDI, *A semi-iterative method for real spectrum singular linear systems with an arbitrary index*, J. Comput. Appl. Math. 87 (1997) 21–38.
[12] R. E. CLINE, *An application of representation of a matrix*, MRC Technical Report 592 (1965).
[13] C. Y. DENG AND H. K. DU, *Representations of the Moore-Penrose inverse for a class of 2-by-2 block operator valued partial matrices*, Linear and Multilinear Algebra 58 (2010) 15–26.
[14] C. Y. DENG AND Y. M. WEI, *A note on the Drazin inverse of an anti-triangular matrix*, Linear Algebra Appl. 431 (2009) 1910–1922.
[15] D. S. DJORDJEVIĆ, Y. M. WEI, *Additive results for the generalized Drazin inverse*, J. Aust. Math. Soc. 73 (2002) 115–125.


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[16] R. G. DOUGLAS, *On majorization, factorization, and range inclusion of operators on Hilbert spaces*, Proc. Amer. Math. Soc. 17 (1966) 413–416.
[17] R. E. HARTE, *Invertibility and Singularity for Bounded Linear Operators*, Marcel Dekker, New York, 1988.
[18] R. E. HARTWIG, X. Z. LI AND Y. M. WEI, *Representations for the Drazin inverse of a $2\times 2$ block matrix*, SIAM J. Matrix Anal. Appl. 27 (2005) 757–771.
[19] R. E. HARTWIG, G. R. WANG AND Y. M. WEI, *Some additive results on Drazin inverse*, Linear Algebra Appl. 322 (2001) 207–217.
[20] L. L. HE, C. N. SONG AND Q. X. XU, *General exact solutions of certain second-order homogeneous algebraic differential equations*, Linear Multilinear Algebra 64 (2016) 1011–1031.
[21] J. J. KOLIHA, *A generalized Drazin inverse*, Glasg. Math. J. 38 (1996) 367–381.
[22] J. J. KOLIHA, V. RAKOČEVIĆ, *Continuity of the Drazin inverse II*, Studia Math. 131 (1998) 167–177.
[23] P. KUNKEL AND V. MEHRMANN, *Differential-algebraic equations (Analysis and numerical solution)*, Zürich: European Mathematical Society Publishing House, 2006.
[24] Y. M. WEI, X. Z. LI AND F. B. BU, *A perturbation bound of the Drazin inverse of a matrix by separation of simple invariant subspaces*, SIAM J. Matrix Anal. Appl. 27 (2005) 72–81.
[25] Q. X. XU, C. N. SONG AND X. F. LIU, *General exact solutions of the second-order homogeneous algebraic differential equations*, Linear Multilinear Algebra 63 (2015) 244–263.
[26] Q. X. XU, C. N. SONG AND L. L. HE, *Representations for the group inverse of anti-triangular block operator matrices*, Linear Algebra Appl. 443 (2014) 191–203.
[27] Q. X. XU, C. N. SONG AND L. L. HE, *Representations for the Drazin inverse of an anti-triangular block operator matrix $E$ with $ind(E)\leqslant 2$*, Linear Multilinear Algebra 66 (2018) 1026–1045.