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SPECTRA OF INFINITE GRAPHS: TWO METHODS OF COMPUTATION

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Abstract. Two methods for computation of the spectra of certain infinite graphs are suggested. The first one can be viewed as a reversed Gram–Schmidt orthogonalization procedure. It relies heavily on the spectral theory of Jacobi matrices. The second method is related to the Schur complement for block matrices. A number of examples including finite graphs with tails, chains of cycles and ladders are worked out in detail.

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