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NEW BEREZIN SYMBOL INEQUALITIES FOR OPERATORS ON THE REPRODUCING KERNEL HILBERT SPACE

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Abstract. We use Kittaneh and Manasrah inequality and Kian's functional calculus method to prove some new inequalities for Berezin symbols and Berezin numbers of operators. In particular, we prove that

$$\operatorname{ber}\left(f(A)^2\right) \leqslant \operatorname{ber}\left(\frac{f(A)^p}{p}+\frac{f(A)^q}{q}\right)$$

for all self-adjoint operators A on the reproducing kernel Hilbert space $\mathscr{H}(\Omega)$ with spectrum in $J \subset (-\infty,+\infty)$ and all continuous nonnegative functions f defined on J. We also prove new upper and lower bounds for Berezin numbers of reproducing kernel Hilbert space operators. Among our results, we prove that if $A:\mathscr{H}(\Omega)\to\mathscr{H}(\Omega)$ is a bounded pseudo-hypornormal operator on the reproducing kernel Hilbert space $\mathscr{H}(\Omega)$, then for all non-negative non-decreasing pseudo-operator convex function f on $[0,\infty)$, we have

$$f(\operatorname{ber}(A)) \leqslant \frac{1}{2}\left\| f\left(\frac{|A|}{1+\frac{\xi^2_{|A|}}{8}}\right)+f\left(\frac{|A^*|}{1+\frac{\xi^2_{|A|}}{8}}\right)\right\|_{\mathrm{Ber}},$$

where $\|.\|_{\mathrm{Ber}}$ denotes the Berezin norm of operator.

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