

L-MATRICES WITH LACUNARY COEFFICIENTS

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Abstract. We show that an L -matrices $A = [a_n]$, with lacunary coefficients (a_n) is a bounded operator on ℓ^2 , provided that (a_n) satisfy an explicit decay rate. Moreover, by a concrete example, we see that the decay restriction is optimal. The extension to operators on ℓ^p spaces, for $p > 1$, is also discussed.

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