

APPROXIMATION OF THE NUMERICAL RANGE OF POLYNOMIAL OPERATOR MATRICES

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Abstract. A linear operator on a Hilbert space may be approximated by finite matrices choosing an orthonormal basis of the Hilbert space. In this paper we establish an approximation of the q -numerical range of a bounded and an unbounded polynomial operator by variational methods. Applications to Hain-Lüst operator and Stokes operator are also given.

Mathematics subject classification (2020): 47A58, 47A20, 47A12, 15A60.

Keywords and phrases: Polynomial operator, q -numerical range, projection method, Hain-Lüst operator, Stokes operator.

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