HARNACK TYPE INEQUALITIES FOR OPERATORS
IN LOGARITHMIC SUBMAJORISATION

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Abstract. This paper studies the Harnack type logarithmic submajorisation and Fuglede–Kadison determinant inequalities for operators in a finite von Neumann algebra. In particular, the Harnack type determinant inequalities due to Lin–Zhang [17] and Yang–Zhang [28] are extended to the case of operators in a finite von Neumann algebra.

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