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SPHERICAL SYMMETRY OF SOME UNITARY INVARIANTS FOR COMMUTING TUPLES

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Abstract. We discuss spherical and Euclidean analogues of joint spectral radius, joint operator norm and joint numerical radius associated with commuting d-tuples of Hilbert space operators. In particular, we deduce their invariance under the action of the group $\mathscr{U}(d)$ of $d \times d$ unitary matrices. Unlike spectral and numerical radii, the analogues of joint operator norm differ in dimension $d > 1$. The joint hyponormality ensures that these analogues of joint operator norm agree in all dimensions. However, the separate hyponormality fails to ensure so.

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