Operators and Matrices
Volume 15, Number 3 (2021), 1189–1199
doi:10.7153/oam-2021-15-75

A NEW MATRIX INEQUALITY INVOLVING PARTIAL TRACES

YONGTAO LI, WEIJUN LIU AND YANG HUANG*

Abstract. Let A be an $m\times m$ positive semidefinite block matrix with each block being n-square. We write tr_1 and tr_2 for the first and second partial trace, respectively. In this paper, we prove the following inequality

$$(\mathrm{tr}A)I_{mn}-(\mathrm{tr}_2A)\otimes I_n \geqslant \pm\big(I_m\otimes(\mathrm{tr}_1A)-A\big).$$

This inequality is not only a generalization of Ando's result [ILAS Conference (2014)] and Lin [Canad. Math. Bull. 59 (2016) 585–591], but it also could be regarded as a complement of a recent result of Choi [Linear Multilinear Algebra 66 (2018) 1619–1625]. Additionally, some new partial traces inequalities for positive semidefinite block matrices are also included.

Mathematics subject classification (2020): 15A45, 15A60, 47B65.
Keywords and phrases: Partial traces, block matrices, positive semidefinite, Cauchy-Khinchin.

REFERENCES

[1] T. ANDO, *Matrix inequalities involving partial traces*, ILAS Conference, 2014.
[2] T. ANDO, *Geometric mean and norm Schwarz inequality*, Ann. Funct. Anal. 7 (2016) 1–8.
[3] R. BHATIA, *Matrix Analysis*, GTM 169, Springer-Verlag, New York, 1997.
[4] R. BHATIA, *Positive Definite Matrices*, Princeton University Press, Princeton, 2007.
[5] M. D. CHOI, *Positive linear maps on C^*-algebras*, Canad. J. Math. 24 (1972) 520–529.
[6] D. CHOI, *Inequalities related to partial transpose and partial trace*, Linear Algebra Appl. 516 (2017) 1–7.
[7] D. CHOI, *Inequalities related to trace and determinant of positive semidefinite block matrices*, Linear Algebra Appl. 532 (2017) 1–7.
[8] D. CHOI, *Inequalities about partial transpose and partial trace*, Linear Multilinear Algebra 66 (2018) 1619–1625.
[9] E. R. VAN DAM, *A Cauchy-Khinchin matrix inequality*, Linear Algebra Appl. 280 (1998) 163–172.
[10] X. FU, P.-S. LAU, T.-Y. TAM, *Linear maps of positive partial transpose matrices and singular value inequalities*, Math. Inequal. Appl. 23 (4) (2020) 1459–1468.
[11] M. GUNUS, J. LIU, S. RAOUAFI, T.-Y. TAM, *Positive semi-definite 2×2 block matrices and norm inequalities*, Linear Algebra Appl. 551 (2018) 83–91.
[12] R. A. HORN, C. R. JOHNSON, *Matrix Analysis, 2nd ed.*, Cambridge University Press, Cambridge, 2013.
[13] F. KITTANEH, M. LIN, *Trace inequalities for positive semidefinite block matrices*, Linear Algebra Appl. 524 (2017) 153–158.
[14] E.-Y. LEE, *The off-diagonal block of a PPT matrix*, Linear Algebra Appl. 486 (2015) 449–453.
[15] Y. LI, Y. HUANG, L. FENG, W. LIU, *Some applications of two completely copositive maps*, Linear Algebra Appl. 590 (2020) 124–132.
[16] M. LIN, *A completely PPT map*, Linear Algebra Appl. 459 (2014) 404–410.
[17] M. LIN, *Inequalities related to 2×2 block PPT matrices*, Oper. Matrices, 9 (2015) 917–924.
[18] M. LIN, *A determinantal inequality involving partial traces*, Canad. Math. Bull. 59 (2016) 585–591.
[19] D. PETZ, *Quantum Information Theory and Quantum Statistics. Theoretical and Mathematical Physics*, Springer, Berlin, 2008.

Paper OaM-15-75

[20] P. ZHANG, *On some inequalities related to positive block matrices*, Linear Algebra Appl. 576 (2019) 258–267.
[21] X. ZHAN, *Matrix Inequalities*, Springer, New York, 2002.
[22] X. ZHAN, *Matrix Theory*, Graduate Studies in Mathematics, vol. 147, Amer. Math. Soc., Providence, RI, 2013.
[23] F. ZHANG, *Matrix Theory: Basic Results and Techniques, 2nd edition*, Springer, New York, 2011.
[24] F. ZHANG, *Positivity of matrices with generalized matrix functions*, Acta Math. Sin. (Engl. Ser.) 28 (2012) 1779–1786.