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ON GENERALIZED DAVIS–WIELANDT RADIUS INEQUALITIES OF SEMI–HILBERTIAN SPACE OPERATORS

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Abstract. Let A be a positive (semidefinite) operator on a complex Hilbert space $\mathscr{H}$ and let $\mathbb{A} = \begin{pmatrix} A & O \\ O & A \end{pmatrix}$. We obtain upper and lower bounds for the A-Davis-Wielandt radius of semi-Hilbertian space operators, which generalize and improve on the existing ones. Further, we derive upper bounds for the A-Davis-Wielandt radius of the sum of the product of semi-Hilbertian space operators. We also obtain upper bounds for the $\mathbb{A}$-Davis-Wielandt radius of 2×2 operator matrices. Finally, we determine the exact value for the $\mathbb{A}$-Davis-Wielandt radius of two operator matrices $\begin{pmatrix} I & X \\ O & O \end{pmatrix}$ and $\begin{pmatrix} O & X \\ O & O \end{pmatrix}$, where X is a semi-Hilbertian space operator, and I, O are the identity operator, the zero operator on $\mathscr{H}$, respectively.

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