

WEIGHTED OPERATOR-VALUED FUNCTION SPACES APPLIED TO THE STABILITY OF DELAY SYSTEMS

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Abstract. This paper extends the theory of Zen spaces (weighted Hardy/Bergman spaces on the right-hand half-plane) to the Hilbert-space valued case, and describes the multipliers on them; it is shown that the methods of H^∞ control can therefore be extended to a family of weighted L^2 input and output spaces. Next, the particular case of retarded delay systems with operator-valued transfer functions is analysed, and the dependence of H^∞ structure on the delay is determined by developing an extension of the Walton–Marshall technique used in the scalar case. The method is illustrated with examples.

Mathematics subject classification (2020): 30H10, 30H20, 44A10, 46E15, 47N70, 93B36.

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REFERENCES

- [1] W. ARENDT, C. J. K. BATTY, M. HIEBER AND F. NEUBRANDER, *Vector-valued Laplace transforms and Cauchy problems*, Second edition, Monographs in Mathematics, 96, Birkhäuser/Springer Basel AG, Basel, 2011.
- [2] R. BELLMAN AND K. L. COOKE, *Differential-difference equations*, Academic Press, New York-London, 1963.
- [3] C. BONNET AND J. R. PARTINGTON, *Analysis of fractional delay systems of retarded and neutral type*, Automatica J. IFAC 38 (2002), no. 8, 1133–1138.
- [4] J. BRAM, *Subnormal operators*, Duke Math. J. 22 (1955), 75–94.
- [5] J. B. CONWAY, *Subnormal operators*, Research Notes in Mathematics, 51. Pitman (Advanced Publishing Program), Boston, Mass. London, 1981.
- [6] B. A. FRANCIS, *A course in H_∞ control theory*, Lecture Notes in Control and Information Sciences, Springer-Verlag, Berlin, 1987.
- [7] Z. HARPER, *Boundedness of convolution operators and input-output maps between weighted spaces*, Complex Anal. Oper. Theory 3 (2009), no. 1, 113–146.
- [8] Z. HARPER, *Laplace transform representations and Paley–Wiener theorems for functions on vertical strips*, Doc. Math. 15 (2010), 235–254.
- [9] R. A. HORN AND C. R. JOHNSON, *Matrix analysis*, Corrected reprint of the 1985 original, Cambridge University Press, Cambridge, 1990.
- [10] B. JACOB, A. MIRONCHENKO, J. R. PARTINGTON AND F. WIRTH, *Noncoercive Lyapunov functions for input-to-state stability of infinite-dimensional systems*, SIAM J. Control. Optim. 58 (2020), no. 5, 2952–2978.
- [11] B. JACOB, J. R. PARTINGTON AND S. POTT, *On Laplace–Carleson embedding theorems*, J. Funct. Anal. 264 (2013), no. 3, 783–814.
- [12] B. JACOB, J. R. PARTINGTON AND S. POTT, *Applications of Laplace–Carleson embeddings to admissibility and controllability*, SIAM J. Control Optim. 52 (2014), no. 2, 1299–1313.
- [13] J. R. PARTINGTON, *Linear operators and linear systems. An analytical approach to control theory*, London Mathematical Society Student Texts, 60, Cambridge University Press, Cambridge, 2004
- [14] V. I. PAULSEN AND M. RAGHUPATHI, *An introduction to the theory of reproducing kernel Hilbert spaces*, Cambridge Studies in Advanced Mathematics, 152.

- [15] C. R. PUTNAM, *The spectra of subnormal operators*, Proc. Amer. Math. Soc. 28 (1971), 473–477.
- [16] M. VIDYASAGAR, *Control System Synthesis: A Factorization Approach*, MIT Press Series in Signal Processing, Optimization and Control, 7, 1985.
- [17] K. WALTON AND J. E. MARSHALL, *Direct method for TDS stability analysis*, IEE proceedings D, control theory and applications 134 (1987), 101–107.
- [18] K. ZHOU, J. C. DOYLE AND K. GLOVER, *Robust and Optimal Control*, Prentice-Hall, Inc., Upper Saddle River, 1996.