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ON THE STAMPFLI POINT OF SOME OPERATORS AND MATRICES

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Abstract. The center of mass of an operator A (denoted $\mathrm{St}(A)$, and called in this paper as the *Stampfli point* of A) was introduced by Stampfli in his Pacific J. Math (1970) paper as the unique $\lambda \in \mathbb{C}$ delivering the minimum value of $\|A - \lambda I\|$. We derive some results concerning the location of $\mathrm{St}(A)$ for several classes of operators, including 2-by-2 block operator matrices with scalar diagonal blocks and 3-by-3 matrices with repeated eigenvalues. We also show that for almost normal A its Stampfli point lies in the convex hull of the spectrum, which is not the case in general. Some relations between the property $\mathrm{St}(A) = 0$ and Roberts orthogonality of A to the identity operator are established.

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