REVISITING THE GRÜSS INEQUALITY

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Abstract. In this article, we explore the celebrated Grüss inequality, where we present a new approach using the Grüss inequality to obtain new refinements of operator means inequalities. We also present several operator Grüss-type inequalities with applications to the numerical radius and entropies.

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