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CHARACTERIZATIONS OF ELEMENTARY OPERATORS

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Abstract. Let $\mathscr{A}$ be an ultraprime algebra and $\mathscr{I}$ a closed ideal in $\mathscr{A}$ with left (resp. right) approximate unit. We characterize elementary operators on $\mathscr{A}$ in terms of their images. We show that if Φ is an elementary operator on $\mathscr{A}$, then the set $\Phi(\mathscr{A}_1)$ (where $\mathscr{A}_1$ is the unit ball of $\mathscr{A}$) is a left (resp. right) uniformly approximable subset of $\mathscr{I}$ if and only if for any minimal length representation $\sum_{i=1}^{k} M_{a_i,b_i}$ of Φ we have $\{a_i\}_{i=1}^{k} \subseteq \mathscr{I}$ (resp. $\{b_i\}_{i=1}^{k} \subseteq \mathscr{I}$).

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