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A FUNCTIONAL DECOMPOSITION OF FINITE BANDWIDTH REPRODUCING KERNEL HILBERT SPACES

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Abstract. In this work, we consider "finite bandwidth" reproducing kernel Hilbert spaces which have orthonormal bases consisting of certain polynomials. We provide general conditions based on a matrix recursion that guarantee such spaces contain a functional multiple of the Hardy space. In a particular case, we obtain an explicit functional decomposition of these spaces that greatly generalizes a previous result in the tridiagonal case due to Adams and McGuire. We also prove that multiplication by z is a bounded operator on these spaces and that they contain the polynomials.

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