

MULTILINEAR HAUSDORFF OPERATOR AND COMMUTATORS ON WEIGHTED MORREY AND HERZ SPACES

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Abstract. In this paper, we establish some necessary and sufficient conditions for the boundedness of multilinear Hausdorff operators on weighted central Morrey and Herz type spaces. Moreover, we also discuss some sufficient conditions for the boundedness of commutators of multilinear Hausdorff operators on weighted Morrey–Herz spaces. By these, we generalize some previous known results.

Mathematics subject classification (2020): 42B35, 46E30, 47B38.

Keywords and phrases: Multilinear operator, Hausdorff operator, Hardy operator, Morrey spaces, Herz spaces, commutators.

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